Matching the quark model to the 1/Nc expansion

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Example: The masses for L=1

Mass operator : $H_{mass} = \sum c_i O_i$ power counting natural size

Building blocks for the mass operator:

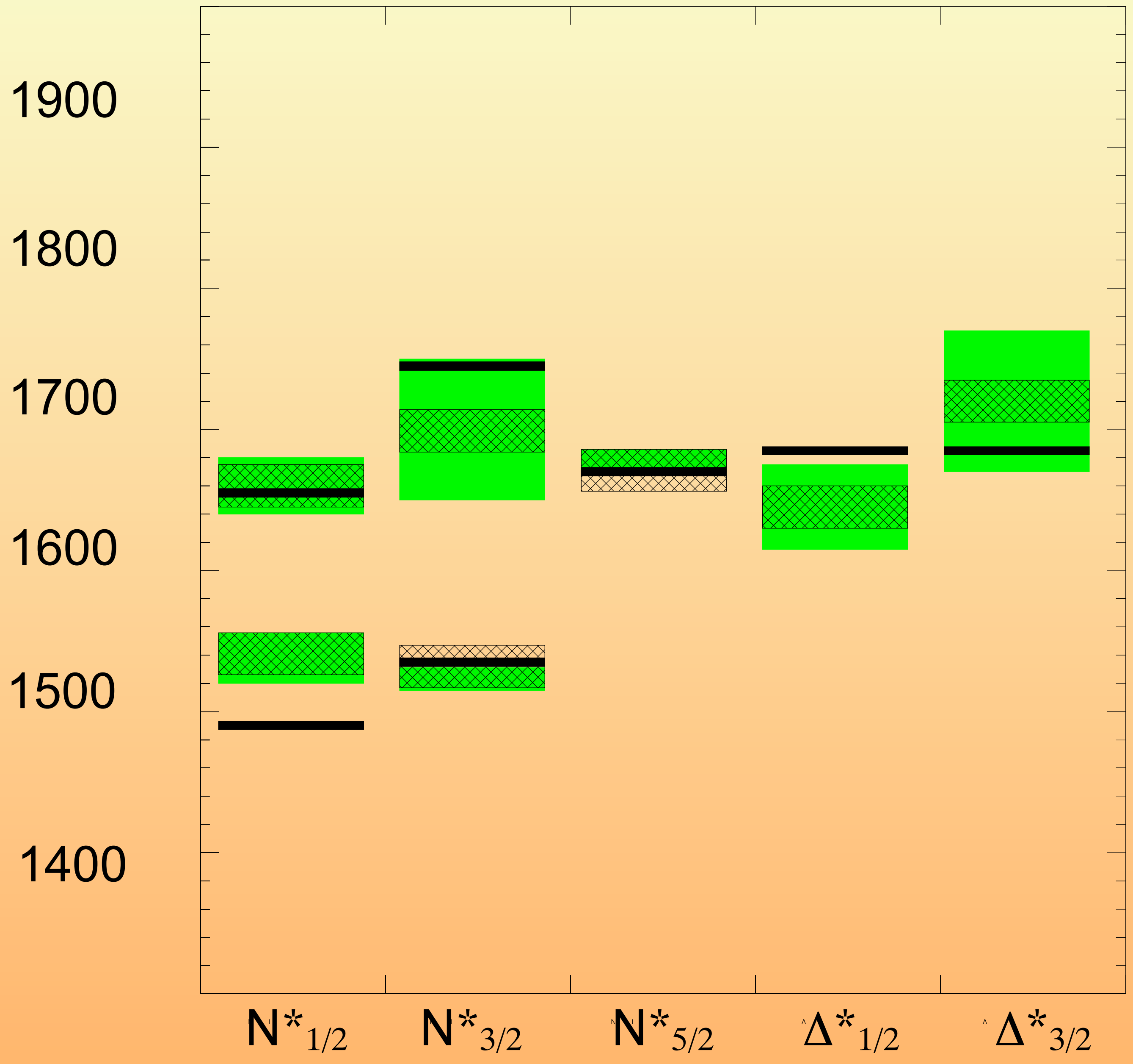
SU(4) generators acting on the core

$$S_c^i = \sum_{\alpha=1}^{N_c-1} s^i_{(\alpha)}, \quad T_c^a = \sum_{\alpha=1}^{N_c-1} t^a_{(\alpha)}, \quad G_c^{ia} = \sum_{\alpha=1}^{N_c-1} s^i_{(\alpha)} t^a_{(\alpha)}$$

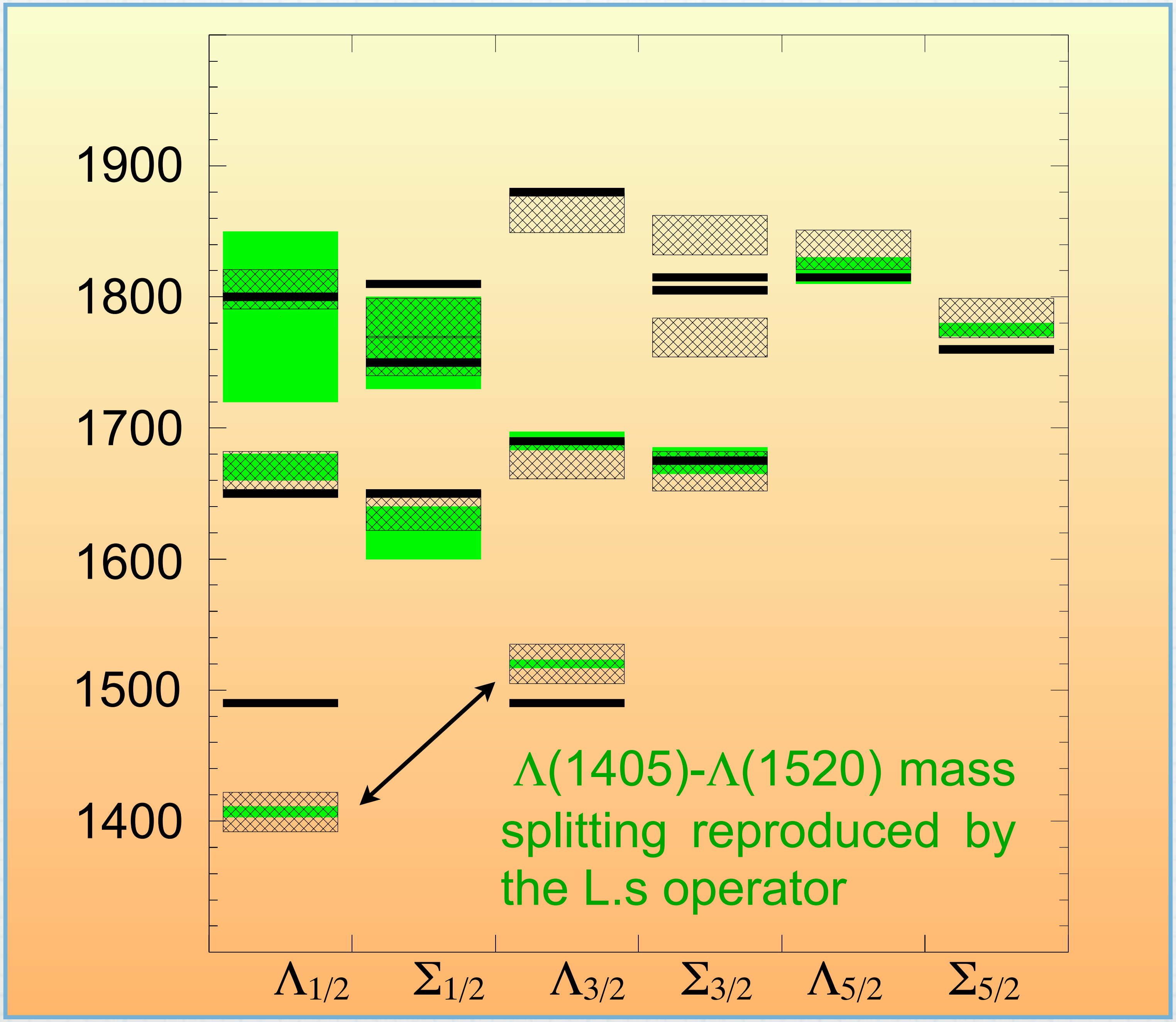
and on the excited quark

s^i, t^a, g^{ia} spin-flavor $\quad$ l^i orbital degrees of freedom

Including 1/Nc and SU(3)



Including 1/Nc and SU(3)



C.S., J.L.Goity and N.N.Scoccola, PRL88, 102002 (2002).

flavor interactions

	Operator	Fitted coef. [MeV]
	$O_1 = N_c\ 1$	$c_1 =$ 449 ± 2
O(1)	$O_2 = l_h\ s_h$	$c_2 =$ 52 ± 15
	$O_3 = \frac{3}{N_c}\ l^{(2)}_{hk}\ g_{ha}\ G^c_{ka}$	$c_3 =$ 116 ± 44
	$O_4 = \frac{4}{N_c+1}\ l_h\ t_a\ G^c_{ha}$	$c_4 =$ 110 ± 16
O(1/Nc)	$O_5 = \frac{1}{N_c}\ l_h\ S^c_h$	$c_5 =$ 74 ± 30
	$O_6 = \frac{1}{N_c}\ S^c_h\ S^c_h$	$c_6 =$ 480 ± 15
	$O_7 = \frac{1}{N_c}\ s_h\ S^c_h$	$c_7 =$ -159 ± 50
	$O_8 = \frac{1}{N_c}\ l^{(2)}_{hk} s_h\ S^c_k$	$c_8 =$ 6 ± 110
	$O_9 = \frac{1}{N_c^2}\ l_h\ g_{ka}\{S^c_k, G^c_{ha}\}$	$c_9 =$ 213 ± 153
	$O_{10} = \frac{1}{N_c^2} t_a\{S^c_h, G^c_{ha}\}$	$c_{10} =$ -168 ± 56
	$O_{11} = \frac{1}{N_c^2}\ l_h\ g_{ha}\{S^c_k, G^c_{ka}\}$	$c_{11} =$ -133 ± 130
SU(3) breaking	$\bar{B}_1 = t_8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 =$ -81 ± 36
	$\bar{B}_2 = T^c_8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 =$ -194 ± 17
	$\bar{B}_3 = \frac{1}{N_c}\ d_{8ab}\ g_{ha}\ G^c_{hb} + \frac{N_c^2-9}{16\sqrt{3}N_c^2(N_c-1)} O_1 + \frac{1}{4\sqrt{3}(N_c-1)} O_6 + \frac{1}{12\sqrt{3}} O_7$	$d_3 =$ -150 ± 301
	$\bar{B}_4 = l_h\ g_{h8} - \frac{1}{2\sqrt{3}} O_2$	$d_4 =$ -82 ± 57

The Isgur-Karl model revisited

L.Galeta, D.Pirjol, C.S., Phys.Rev. D80, 116004 (2009)

confining potential + hyperfine interaction:

$$\mathcal{H}_{IK} = H_0 + \mathcal{H}_{\rm hyp}$$

$$\mathcal{H}_{\rm hyp} = A \sum_{i<j} \Big[\frac{8\pi}{3} \vec{s}_i \cdot \vec{s}_j \delta^{(3)}(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} (3\vec{s}_i \cdot \hat{r}_{ij} \ \vec{s}_j \cdot \hat{r}_{ij} - \vec{s}_i \cdot \vec{s}_j) \Big]$$

Matching: find c_i and O_i

$$\boxed{\langle B|\mathcal{H}_{\rm hyp}|B\rangle = \sum_i c_i \langle \Phi(JSI)|O_i|\Phi(JSI)\rangle}$$

IK model matching

$$\langle B|\mathcal{H}_{\mathrm{hyp}}|B\rangle = \sum_i c_i \langle \Phi(JSI)|O_i|\Phi(JSI)\rangle$$

The result of the matching is

$$\boxed{\hat{M} = c_0 + aS_c^2 + bL_2^{ab}\{S_c^a\,, S_c^b\} + cL_2^{ab}\{s_1^a\,, S_c^b\}}$$

$$\begin{aligned} a &= \frac{1}{2}\langle R_S\rangle\,, \\ b &= \frac{1}{12}\langle Q_S\rangle - \frac{1}{6}\langle Q_{MS}\rangle\,, \\ c &= \frac{1}{6}\langle Q_S\rangle + \frac{1}{6}\langle Q_{MS}\rangle\,. \end{aligned}$$

for a contact interaction:

$$\langle R_{MS}\rangle = -\langle R_S\rangle$$

spatial integrals

The operators that appear are O_1, O_6, O_8, O_{17}

IK mass matrix

$$
\begin{aligned}
M_{1/2} &= \begin{pmatrix} c_0 + a & -\frac{5}{3}b + \frac{5}{6}c \\ -\frac{5}{3}b + \frac{5}{6}c & c_0 + 2a + \frac{5}{3}(b+c) \end{pmatrix}, \\
M_{3/2} &= \begin{pmatrix} c_0 + a & \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c \\ \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c & c_0 + 2a - \frac{4}{3}(b+c) \end{pmatrix}, \\
M_{5/2} &= c_0 + 2a + \frac{1}{3}(b+c)\,, \\
\Delta_{1/2} &= \Delta_{3/2} = c_0 + 2a\,.
\end{aligned}
$$

The spatial integrals can be computed explicitly in the original IK model with harmonic confinement:

$$
a = \frac{1}{2}\delta\,, \qquad b = \frac{1}{20}\delta\,, \qquad c = -\frac{1}{5}\delta\,.
$$

$$\delta = M_\Delta - M_N \sim 300 \text{ MeV}$$

ground states

One gluon exchange model

The QCD Breit-Wigner interaction

$$\mathcal{H} = \sum_{i<j} V_{ij} \qquad\qquad V = V_{ss} + V_q + V_{so}$$

$$V_{ss} = \frac{16\pi\alpha_s}{9}\frac{1}{m_i m_j}\vec{s}_i\cdot\vec{s}_j\delta(\vec{r}_{ij})$$ Spin-spin

$$V_q = \frac{2\alpha_s}{3r_{ij}^3}\frac{1}{m_i m_j}\left[\frac{3}{r_{ij}^2}(\vec{r}_{ij}\cdot\vec{s}_i)(\vec{r}_{ij}\cdot\vec{s}_j) - (\vec{s}_i\cdot\vec{s}_j)\right]$$ Quadrupole

$$V_{so} = \frac{\alpha_s}{3r_{ij}^3}\left[\frac{1}{m_i^2}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_i - \frac{1}{m_j^2}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j\right.$$ Spin-orbit

$$\left. + \frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j - \frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_i\right]$$

One gluon exchange

more general

$$V_{ss} = \sum_{i<j=1}^{N} f_0(r_{ij}) \vec{s}_i \cdot \vec{s}_j$$

$$V_q = \sum_{i<j=1}^{N} f_2(r_{ij}) \Big[3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \Big]$$

$$V_{so} = \sum_{i<j=1}^{N} f_1(r_{ij}) \Big[(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \\ + 2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \Big]$$

One pion exchange

$$V_{ss} = \sum_{i<j=1}^{N} g_0(r_{ij}) \vec{s}_i \cdot \vec{s}_j t_i^a t_j^a$$

$$V_q = \sum_{i<j=1}^{N} g_2(r_{ij}) \Big[3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \Big] t_i^a t_j^a$$

$$V_{so} = \sum_{i<j=1}^{N} g_1(r_{ij}) \Big[(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j$$

$$+2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \Big] t_i^a t_j^a$$

Consider all possible 2-body interactions

D.Pirjol, C.S., Phys.Rev.Lett.102:152002(2009)

The most general 2-body quark interaction

Complete list of 2-body spin-flavor quark operators, and their projection onto irreps of $S_3^{\mathrm{sp-fl}}$

Operator	$\mathcal{O}_{ij}$	O_S	O_{MS}
Scalar	1	1	$-$
	$t_i^a t_j^a$	$T^2 - 3C_2(F)$	$T^2 - 3t_1 T_c - 3C_2(F)$
	$\vec{s}_i \cdot \vec{s}_j$	$\vec{S}^2 - \frac{9}{4}$	$\vec{S}^2 - 3\vec{s}_1 \cdot \vec{S}_c - \frac{9}{4}$
	$\vec{s}_i \cdot \vec{s}_j t_i^a t_j^a$	$G^2 - \frac{9}{4}C_2(F)$	$3g_1 G_c - G^2 + \frac{9}{4}C_2(F)$
Vector (symm)	$\vec{s}_i + \vec{s}_j$	$\vec{L} \cdot \vec{S}$	$3\vec{L} \cdot \vec{s} - \vec{L} \cdot \vec{S}$
	$(\vec{s}_i + \vec{s}_j) t_i^a t_j^a$	$\frac{1}{2}L^i\{G^{ia}, T^a\} - C_2(F)L^i S^i$	$-\frac{1}{2}L^i S_c^i + L^i g_1^{ia} T_c^a + L^i t_1^a G_c^{ia}$
Vector (anti)	$\vec{s}_i - \vec{s}_j$	$-$	$3\vec{L} \cdot \vec{s} - \vec{L} \cdot \vec{S}$
	$(\vec{s}_i - \vec{s}_j) t_i^a t_j^a$	$-$	$L^i g_1^{ia} T_c^a - L^i t_1^a G_c^{ia}$
Tensor (symm)	$\{s_i^a, s_j^b\}$	$L_2^{ij}\{S^i, S^j\}$	$3L_2^{ij}\{s^i, S_c^j\} - L_2^{ij}\{S^i, S^j\}$
	$\{s_i^a, s_j^b\} t_i^c t_j^c$	$L_2^{ij}\{G^{ia}, G^{ja}\}$	$L_2^{ij} g_1^{ia} G_c^{ja} - \frac{1}{4}L_2^{ij}\{S_c^i, S_c^j\}$
Tensor (anti)	$[s_i^a, s_j^b]$	$-$	0
	$[s_i^a, s_j^b] t_i^c t_j^c$	$-$	0

$$C_2(F) = \frac{F^2 - 1}{2F}$$

[Contribute on S, MS states] [Contribute on MS states]

Complete basis of 1/Nc operators induced by 2-body quark operators

spin-spin

$$O_1 = T^2$$

$$O_2 = \vec{S}_c^2$$

$$O_3 = \vec{s}_1 \cdot \vec{S}_c$$

spin-orbit

$$O_4 = \vec{L} \cdot \vec{S}_c$$

$$O_5 = \vec{L} \cdot \vec{s}_1$$

$$O_6 = L^i t_1^a G_c^{ia}$$

$$O_7 = L^i g_1^{ia} T_c^a$$

tensor

$$O_8 = L_2^{ij} \{S_c^i, S_c^j\}$$

$$O_9 = L_2^{ij} s_1^i S_c^j$$

$$O_{10} = L_2^{ij} g_1^{ia} G_c^{ja}$$

Hadronic mass matrix (L=1 states)

$$N_i^* = \sum_{i=0}^{10} \hat{M}_{ij} c_j$$

$$N_i^* = (N_{1/2}, N'_{1/2}, N_{1/2}-N'_{1/2}, N_{3/2}, N'_{3/2}, N_{3/2}-N'_{3/2}, N_{5/2}, \Delta_{1/2}, \Delta_{3/2}, \Lambda_{1/2}, \Lambda_{3/2})$$

Valid for any model with only 2-body quark interactions.

The rank of M is 9, this leads to 2 mass relations:

Two new mass relations

#1 requires only isospin symmetry

$$\frac{1}{2}(N(1535)+N(1650))+\frac{1}{2}(N(1535)-N(1650))(3\cos 2\theta_{N1}+\sin 2\theta_{N1})$$
$$-\frac{7}{5}(N(1520)+N(1700))+(N(1520)-N(1700))\Big[-\frac{3}{5}\cos 2\theta_{N3}+\sqrt{\frac{5}{2}}\sin 2\theta_{N3}\Big]=-2\Delta_{1/2}+2\Delta_{3/2}-\frac{9}{5}N_{5/2}$$

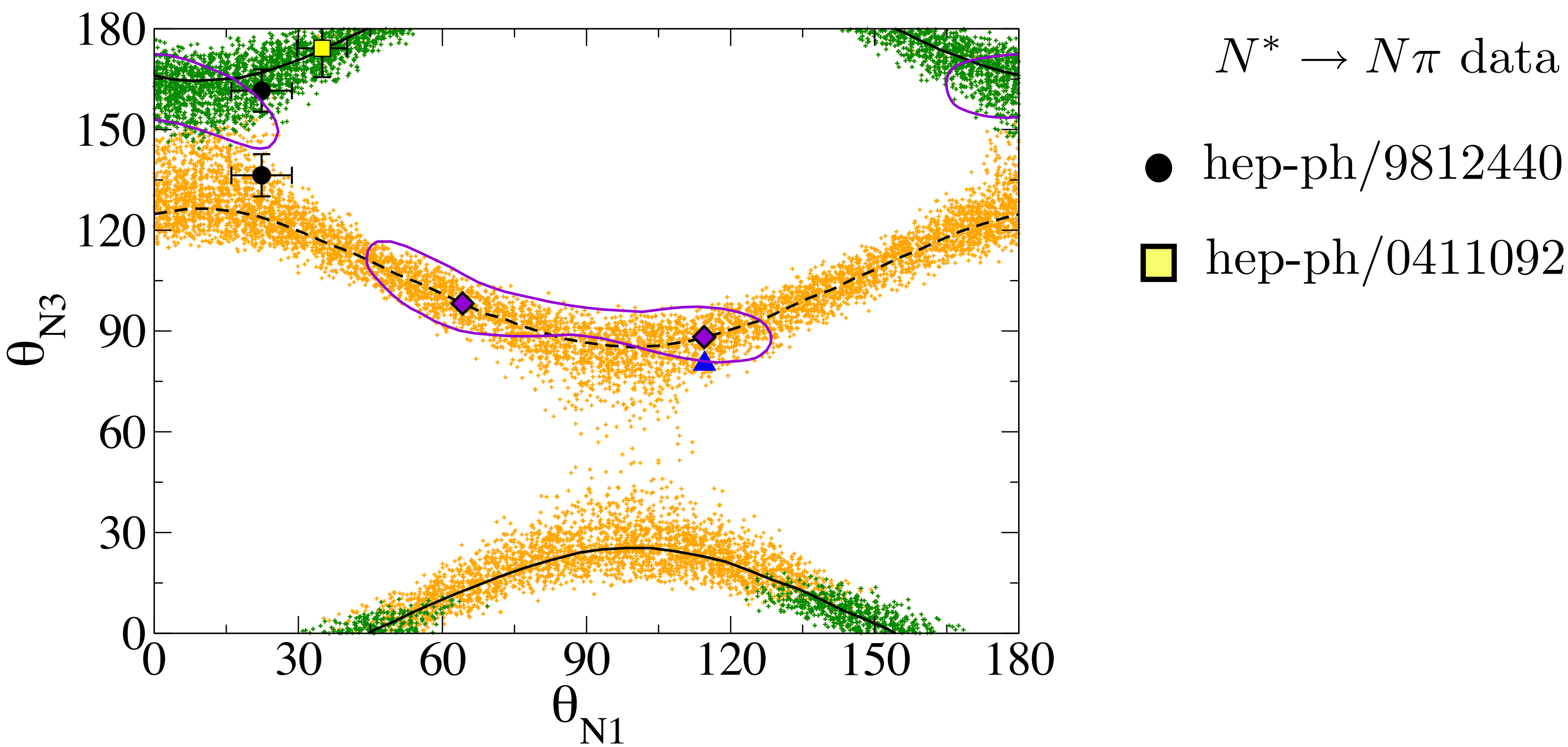
#2 includes SU(3) breaking

$$\begin{aligned}\bar{\Lambda} \;=\; & \frac{1}{6}(N(1535)+N(1650))+\frac{17}{15}(N(1520)+N(1700))-\frac{3}{5}N_{5/2}(1675)-\Delta_{1/2}(1620)\\ & -\frac{1}{6}(N(1535)-N(1650))(\cos 2\theta_{N1}+\sin 2\theta_{N1})+(N(1520)-N(1700))(\frac{13}{15}\cos 2\theta_{N3}-\frac{1}{3}\sqrt{\frac{5}{2}}\sin 2\theta_{N3})\end{aligned}$$

First universal relation

Correlation between mixing angles in the J=1/2 and 3/2 sectors

Requires only isospin symmetry



- Compare with direct determinations of the mixing angles -> test for the presence of 3-body quark interactions

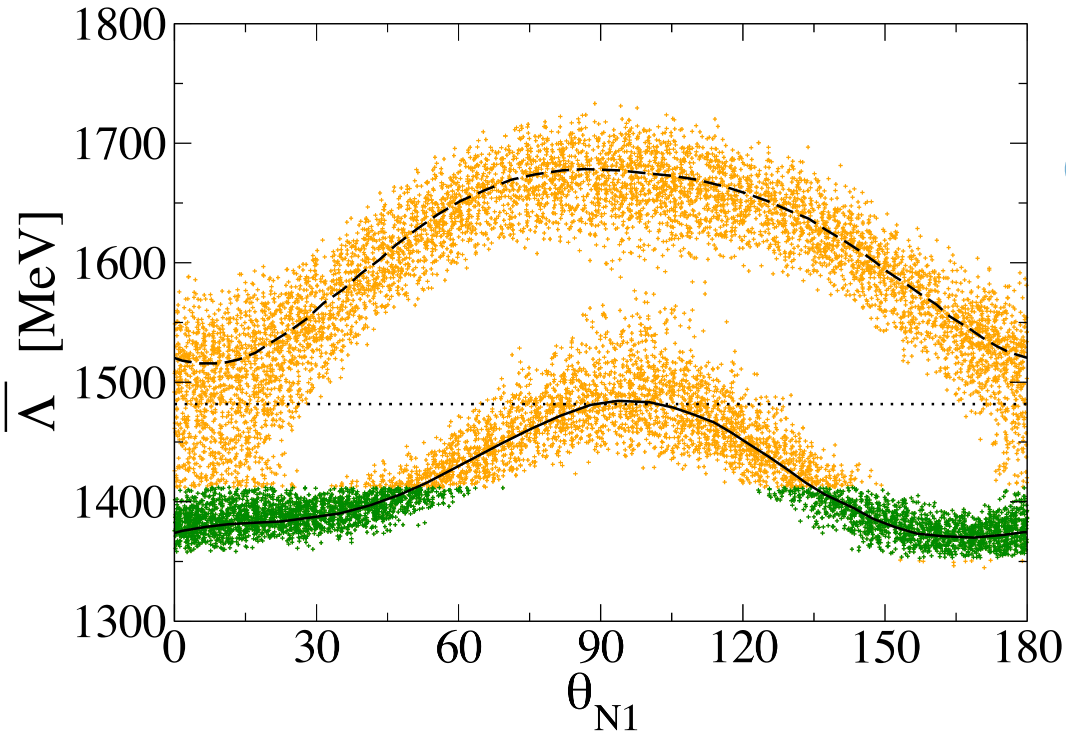
- Tests for specific models of 2-body quark interactions, e.g. OGE model

Second universal relation

Expresses the spin-weighted SU(3) singlet mass

Takes into account SU(3) symmetry breaking

$$\bar{\Lambda} = \frac{1}{3}\Lambda_{1/2} + \frac{2}{3}\Lambda_{3/2}$$



$$\langle\Lambda\rangle^{\rm exp}_{\rm av} = 1481 \pm 3.1\ \mathrm{MeV}$$

$$\langle\Lambda\rangle - \langle\Lambda\rangle^{\rm exp} = 100 \pm 30\ \mathrm{MeV}$$

Combining the two universal relations gives a determination of the mixing angles from hadron masses alone

Dynamics

D.Pirjol, C.S.

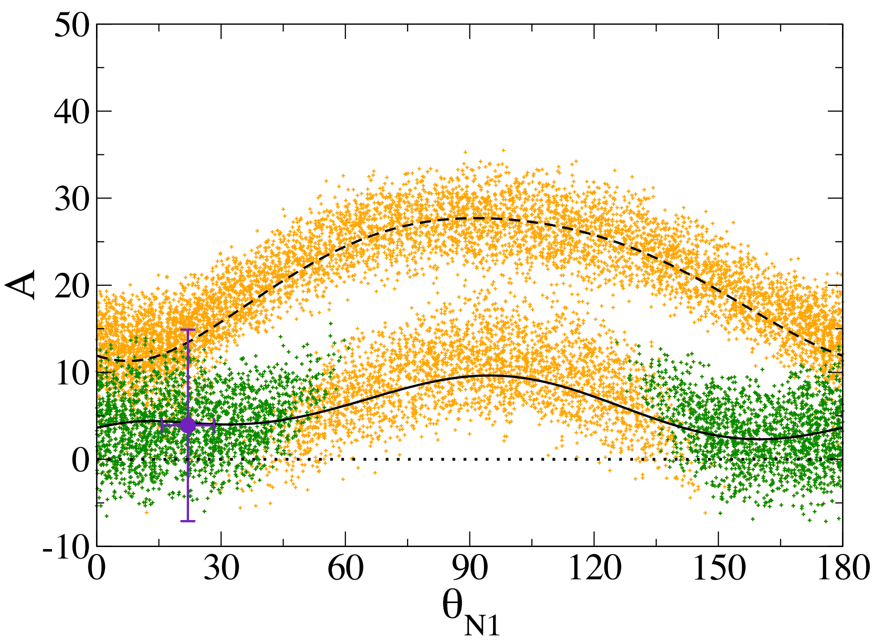
all scalar interactions

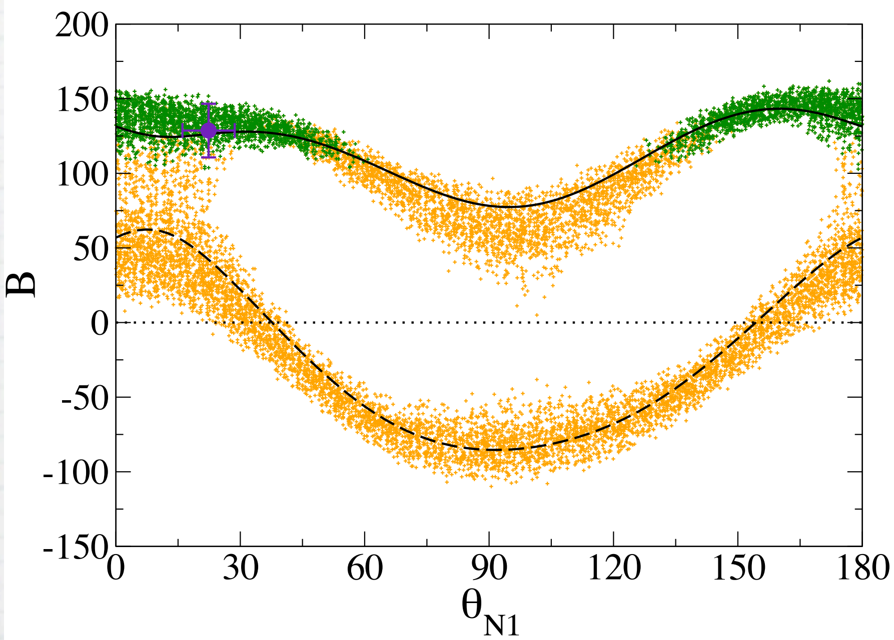
$$V_s = \sum_{i<j} f_0(r_{ij}) \begin{cases} \vec{s}_i \cdot \vec{s}_j & \text{OGE} \\ \vec{s}_i \cdot \vec{s}_j \; t_i^a \cdot t_j^a & \text{OPE} \\ \ldots & \ldots \end{cases}$$

match on

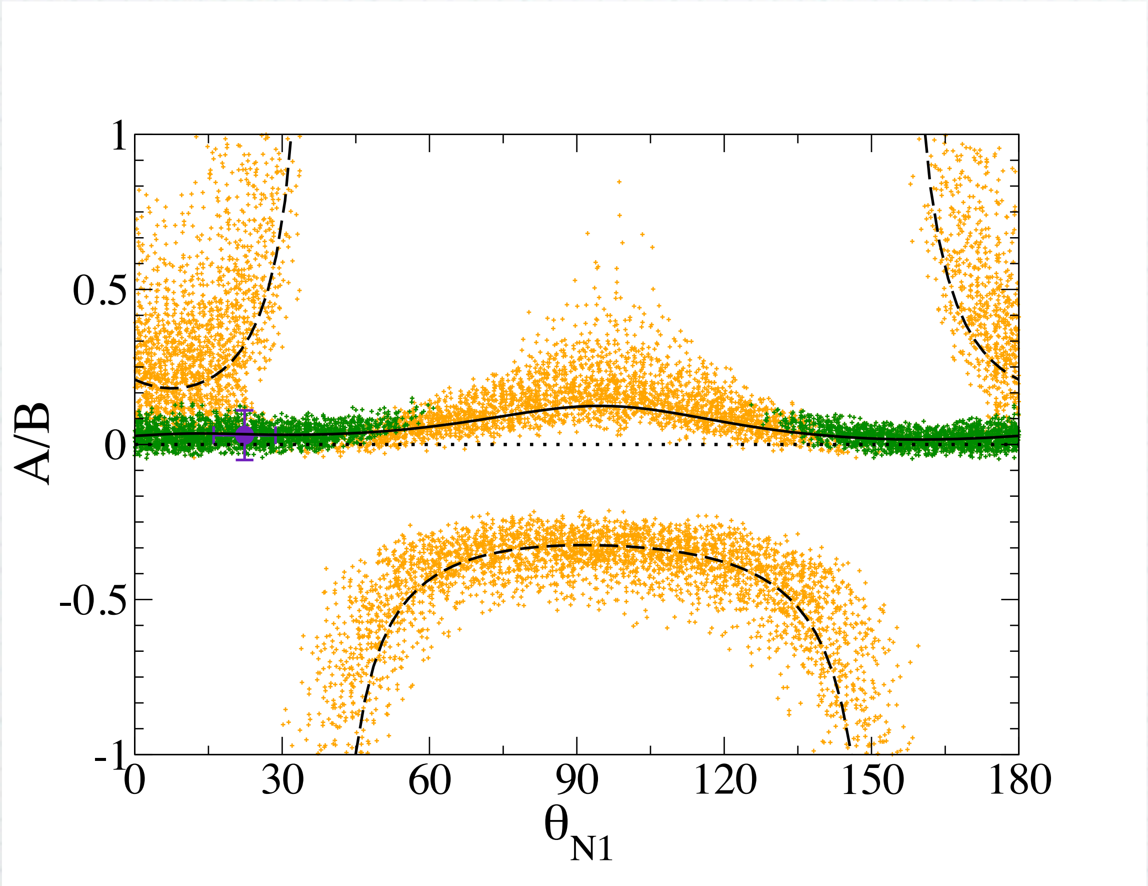
$$M = c_0 + c_1 T^2 + c_2 S_c^2 + c_3 \vec{s}_1 \cdot \vec{S}_c + \cdots$$

$A = c_1 - \frac{1}{2} c_3$ $\qquad$ $B = c_2 + c_3$





Dynamical suppression



A/B = O(1)

A=0 for a **ALL** contact interactions: $f_0(r_{ij}) = \delta(r_{ij})$

A=0 for OPE, even if spread out: $f_0(r_{ij}) \neq \delta(r_{ij})$

In particular, OPE and massless OGE cannot be distinguished here.

Conclusions

* The coefficients of the 1/Nc expansion can be matched to overlap integrals in quark models
* Flavor dependent interactions are important
* Two new mass relations for N*'s that constrain the mixing angles and test for 3-body forces